On the Gaussian Interference Channel with Half-Duplex Causal Cognition

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Abstract

This paper studies the two-user Gaussian interference channel with half-duplex *causal cognition*, or with unilateral source cooperation. This channel model consists of two source-destination pairs sharing a common wireless channel. One of the sources, referred to as the *cognitive transmitter*, can gather information about the activity of the other source, referred to as the *primary transmitter*, through a noisy link and can therefore assist, i.e., causally cooperate, in sending the primary's data. Due to practical constraints, the cognitive transmitter is assumed to work in half-duplex mode, that is, it can not simultaneously transmit and receive. This model is more relevant for practical cognitive radio systems than the classical information theoretic cognitive channel model, where the cognitive transmitter is assumed to have non-causal a priori knowledge of the primary's message. Different network topologies are considered, corresponding to different interference scenarios: (i) the interference-symmetric scenario, where both destinations are in the coverage area of the two sources and therefore experience interference, and (ii) the interference-asymmetric scenario, where one destination does not suffer from interference. For each topology the ultimate sum-rate performance is studied by first deriving the generalized Degrees-of-Freedom (gDoF), or "sum-capacity pre-log" in the high-SNR regime, and then showing relatively simple coding schemes that achieve a sum-rate upper bound to within a constant number of bits for any SNR. Finally the gDoF of the causal cognitive channel is compared to that of the classical non-cooperative interference channel (to assess when cooperation is useless in practice) and to that of the non-causal cognitive channel (to identify the parameter regimes where unilateral causal cooperation attains its ideal ultimate limit).

Index Terms

Cognitive Radio, Causal Unilateral Cooperation, Half-Duplex, Generalized Degrees of Freedom, Inner bound, Outer bound, Constant Gap.

I. Introduction

The next major upgrade of fourth generation cellular networks will consist of a massive deployment of radio infrastructure nodes (i.e., base stations and relay stations) enabling different aspects of the cognitive radio paradigm in an operated network scenario [1]. Cognitive radio concepts, such as centralized and distributed interference management and collaboration between radio nodes, will allow flexible and multi-band access to the spectrum. Radio infrastructure nodes will come in several flavors, characterized primarily by their available bandwidth and number of concurrent frequency channels on which they can simultaneously operate (spectrum aggregation), the capacity of their backhaul links to the operator's core network (wireless, high throughput/low-latency wired interconnect, non carrier-grade wired backhaul, etc.), their ability to collaborate with other similar nodes, and their coverage area and tolerance to interference. The combined use of several infrastructure nodes with different features will result in so-called *heterogenous networks*. The collaborative features of heterogeneous networks can range from fully-centralized Multiple-Input-Multiple-Output (MIMO) with distributed antennas (very high-quality backhaul links for node connection) achieving full use of the network resources, to distributed (causal) MIMO with improved resource utilization compared to point-to-point channels, or to looser forms of collaboration such as joint time/frequency-allocation for improving link-quality.

In this paper we consider a particular aspect of future heterogeneous networks, namely, a practical application of the cognitive overlay paradigm [2]. The channel is modeled as a Gaussian two-user Half-Duplex Causal Cognitive Interference Channel (G-HD-CCIC) where one *cognitive* source, CTx, gathers information about the activity of the other *primary* source, PTx, through a noisy channel. The CTx assists / causally cooperates in sending the PTx's data to the primary receiver, PRx. In the language of [1], the PTx could be a macro-base station with a large coverage area and the CTx a small-cell base station or relay station (indoor or a localized coverage-area); the PTx aims to serve a large-number of users, here for simplicity modeled by a single PRx, which may or may not be in the coverage area of CTx; the CTx aims to serve a smaller-number of users, here modeled for simplicity as a single CRx, which may or may not be in the coverage area of PTx. The link between the two transmitters is noisy and of limited capacity. We assume

that both transmitters have independently generated messages, each known at the corresponding source only. This corresponds to a scenario where PTx and CTx have separate interconnections to the core network or where the schedulers in PTx and CTx operate in a non-coordinated fashion. We further consider the case where the link from PTx to CTx is unilateral so that CTx can *causally* learn the PTx's message but not vice-versa. This models the scenario where the coverage-area of CTx is smaller than that of PTx. The final modeling assumption is that CTx operates in half-duplex mode, e.g., through time- or frequency-division, meaning that in any time/frequency instant the CTx cannot simultaneously transmit and receive.

We study different deployment configurations, which correspond to different interference scenarios. In the interference-symmetric scenario (see Fig. 1, left) both destinations are in the coverage area of the two sources; this implies that both destinations are interfered. In the interference-asymmetric scenario, one destination does not suffer from interference; in this case one of the interfering links is set to zero; due to the asymmetry in the cooperation, two interference-asymmetric scenarios must be considered: the Z-channel, where the link from PTx to CRx is non-existent (i.e., CRx is out of the range of PTx, see Fig. 1, middle) and the S-channel, where the link from CTx to PRx is non-existent (i.e., PRx is out of the range of CTx, see Fig. 1, right).

For each topology we study the ultimate sum-rate performance in the spirit of [3]. We first derive in closed form the generalized Degrees-of-Freedom (gDoF), or "sum-capacity pre-log" in the high-SNR regime, and we then show relatively simple coding schemes that achieve the sum-rate upper bound to within a constant number of bits regardless of the SNR.

In the classical information theoretic cognitive radio overlay paradigm, the CTx is assumed to have non-causal, i.e., before transmission begins, knowledge of the message and codebook of the PTx [4]. The capacity region of this system in Gaussian noise is known exactly for some channel parameters and to within 1 bit otherwise [5]. The assumption of a priori knowledge at the CTx is too idealistic for practical schemes and reasonable only in certain situations, such as when the PTx and the CTx are very close to each other. Motivated by this practical issue, we assume the existence of a noisy link between the two transmitters through which the CTx can causally learn the PTx data. Due to practical constraints, such as the inability to perfectly cancel

the self-interference, the CTx is assumed to work in HD mode. This model is an Interference Channel (IC) with source cooperation [6], more specifically, and IC with unilateral cooperation. As such, the CCIC can be studied within the framework of the IC with source cooperation.

The cooperative IC was first studied in [6], where bilateral Full-Duplex (FD) source or destination cooperation was analyzed by developing capacity outer and inner bounds for the Gaussian noise channel. In [7], a novel sum-rate outer bound for the general memoryless IC with bilateral FD cooperation was derived by extending the idea of [8]. In [9], a novel sum-rate outer bound for the class of FD "injective semi-deterministic" IC was derived by extending the idea of [10]. In [11], a novel sum-rate outer bound based on the dependance-balance idea of [12] was derived. In [13], an achievable scheme for the IC with bilateral FD source cooperation was proposed by exploiting rate-splitting, Dirty-Paper-Coding (DPC) [14] and partial-decode-and-forward [15]. For bilateral FD source cooperation, [9] characterized the sum-capacity to within 20 bits for all channel parameters, while [16] to within 4 bits in the case of "strong cooperation". For unilateral FD source cooperation, [17] characterized the sum-capacity to within 7.3 bits in the interference-symmetric case and to within 1 bit and 4 bits for the Z- and S-channel, respectively.

The case of HD cooperation can be studied as a special case of FD cooperation by using the formalism of [18]. This approach is usually not followed in the literature, often making imprecise claims about capacity and Gaussian capacity to within a constant gap. In [18], it was shown that the largest rate can be achieved by randomly switching between the transmit- and receive-phases at the HD relay node in a simple relay channel. In this way, the randomness that lies into the switch can be harnessed to transmit (at most 1 bit per channel use of) information to the destination. We shall refer to this specific HD mode of operation as *random switch* [18], as opposed to *deterministic switch* where the transmit- and receive-phases are predetermined and therefore known to all nodes. In [19], the author studied the Gaussian IC with bilateral and unilateral HD source cooperation, by characterizing the Gaussian sum-capacity to within 20 bits and 31 bits, respectively, with deterministic switch at the CTx. In this work we overcome a number of limitations and improve on the results of [19] as follows: (i) we properly account for random switch at the CTx in the outer bound by using the formalism of [18] and without

the need to develop a separate theory for the FD and HD cases, (ii) we consider the classical definition of sum-capacity without introducing any “back-off” in the PTx rate (which can be interpreted as a sort of interference margin at the PRx), (iii) we derive the gDoF in closed form rather than expressing it implicitly as the solution of a linear program, and (iv) we reduce the gap to 10.503 bits for the symmetric-interference case, and to 4.507 bits and 5 bits for the Z- and S-channels, respectively, through novel achievable schemes.

As in [19], our ‘optimal to within a constant gap’-achievable schemes are inspired by the Linear Deterministic Approximation (LDA) of the Gaussian noise channel at high SNR. The LDA was first proposed in [20] in the context of the relay networks as a way to capture, in a simple deterministic way, the interaction between interfering signals. In the LDA the effect of the noise is neglected thereby allowing for a complete characterization of the capacity region in many instances where the capacity of the noisy channel counterpart is a long standing open problem. More importantly, the capacity achieving schemes in the LDA model can be ‘translated’ into schemes for the Gaussian noise case that, although not optimal in general, in the worst SNR case are at most a constant and finite number of bits away from an outer bound. LDA-inspired schemes are often rather simple and hence more amenable to practical implementation. For example, the schemes presented here only employ successive decoding rather than joint decoding. As pointed out in [17], there exists a trade-off among complexity and constant gap, i.e., the more complex the achievable schemes the smaller the gap.

Finally, we compare the gDoF of the G-HD-CCIC with that of two well-known scenarios: the classical non-cooperative IC, i.e., where there is no cooperation among the nodes [3], and the non-causal cognitive channel, i.e., where the CTx has non-causal a priori knowledge of the PTx’s message [5]. In particular, we find the parameter regimes where unilateral causal HD cooperation does not yield benefits compared to the non-cooperative IC [3], and those where it attains the ultimate performance limits of the non-causal cognitive radio [5]. Interestingly, we show that in regimes where the G-HD-CCIC outperforms the non-cooperative IC the strength of the cooperation link among the transmitters must be approximately larger than the sum-capacity of the non-cooperative IC.

The rest of the paper is organized as follows. Section II describes the channel model and defines the concept of gDoF and that of sum-capacity to within a constant gap. Section III derives a number of outer-bounds by adapting known FD bounds to the HD case that are then used to derive the gDoF and to characterize the sum-capacity to within a constant gap for the symmetric and asymmetric G-HD-CCIC, in Section IV and Section V, respectively. Section VI concludes the paper. The details of the proofs can be found in Appendix.

II. Channel Model

We use the notation convention of [15]: $[n_1 : n_2]$ is the set of integers from n_1 to $n_2 \geq n_1$, $[x]^+ := \max\{0, x\}$ for $x \in \mathbb{R}$, and Y^j is a vector of length j with component $(Y_1, \ldots, Y_j)$.

A general memoryless CCIC consists of two input alphabets $(\mathcal{X}_\mathsf{p}, \mathcal{X}_\mathsf{c})$, three outputs alphabets $(\mathcal{Y}_\mathsf{f}, \mathcal{Y}_\mathsf{p}, \mathcal{Y}_\mathsf{c})$ and a memoryless transition probability $\mathbb{P}_{\mathsf{Y_f},\mathsf{Y_p},\mathsf{Y_c}|\mathsf{X_p},\mathsf{X_c}}$. PTx has a message $\mathsf{W_p} \in [1 : 2^{N\mathsf{R_p}}]$ for PRx and CTx aims to transmit a message $\mathsf{W_c} \in [1 : 2^{N\mathsf{R_c}}]$ to CRx, where N denotes the codeword length and $\mathsf{R_p}$ and $\mathsf{R_c}$ are the transmission rates for PTx and CTx, respectively, measured in bits per channel use (logarithms are in base 2). The messages $\mathsf{W_p}$ and $\mathsf{W_c}$ are independent and uniformly distributed on their respective domains. At time t, $t \in [1 : N]$, PTx maps its message $\mathsf{W_p}$ into a channel input symbol according to $\mathsf{X}_{\mathsf{p},t}(\mathsf{W_p})$ and CTx encodes its message $\mathsf{W_c}$ and its past channel observations into $\mathsf{X}_{\mathsf{c},t}\left(\mathsf{W_c}, \mathsf{Y}_\mathsf{f}^{t-1}\right)$. At time N, PRx outputs an estimate of $\mathsf{W_p}$ based on all its channel observations as $\widehat{\mathsf{W}}_\mathsf{p}\left(\mathsf{Y}_\mathsf{p}^N\right)$ and similarly CRx outputs $\widehat{\mathsf{W}}_\mathsf{c}\left(\mathsf{Y}_\mathsf{c}^N\right)$. The capacity region is defined as the convex closure of all non-negative rate pairs $(\mathsf{R_p}, \mathsf{R_c})$ such that $\max_{i \in \{\mathsf{p},\mathsf{c}\}} \mathbb{P}\left[\widehat{\mathsf{W}}_i \neq \mathsf{W}_i\right] \to 0$ as $N \to +\infty$.

In this general memoryless framework, CTx can simultaneously send and receive, i.e., it operates in FD mode. HD channels represent a special case of the memoryless FD framework as pointed out in [18]. With a slight abuse of notation, we let the channel input at the CTx be the pair $(\mathsf{X_c}, \mathsf{M_c})$, where $\mathsf{X_c} \in \mathcal{X}_\mathsf{c}$ as before and $\mathsf{M_c} \in \{0, 1\}$ represents the state random variable that indicates whether the CTx is in receive-mode ($\mathsf{M_c} = 0$) or in transmit-mode ($\mathsf{M_c} = 1$). Thus, the memoryless HD channel transition probability is defined by $\mathbb{P}_{\mathsf{Y_f},\mathsf{Y_p},\mathsf{Y_c}|\mathsf{X_p},\mathsf{X_c},\mathsf{M_c}=0} = \mathbb{P}^{(0)}_{\mathsf{Y_f},\mathsf{Y_p},\mathsf{Y_c}|\mathsf{X_p},\mathsf{M_c}=0}$ and $\mathbb{P}_{\mathsf{Y_f},\mathsf{Y_p},\mathsf{Y_c}|\mathsf{X_p},\mathsf{X_c},\mathsf{M_c}=1} = \mathbb{P}^{(1)}_{\mathsf{Y_p},\mathsf{Y_c}|\mathsf{X_p},\mathsf{X_c},\mathsf{M_c}=1}\mathbb{P}^{(1)}_{\mathsf{Y_f}|\mathsf{M_c}=1}$, i.e., when the CTx is in receive-mode ($\mathsf{M_c} =$

0) the outputs $\mathsf{Y_f}, \mathsf{Y_p}, \mathsf{Y_c}$ are independent of $\mathsf{X_c}$ and when the CTx is in transmit-mode ($\mathsf{M_c} = 1$) the output $\mathsf{Y_f}$ is independent of everything else.

A single-antenna G-HD-CCIC is described by the input/output relationship

$$\begin{bmatrix} \mathsf{Y_f} \\ \mathsf{Y_p} \\ \mathsf{Y_c} \end{bmatrix} = \begin{bmatrix} 1-\mathsf{M_c} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{\mathsf{C}} & \star \\ \sqrt{\mathsf{S_p}} & \sqrt{\mathsf{I_p}}\, e^{j\theta_\mathsf{p}} \\ \sqrt{\mathsf{I_c}}\, e^{j\theta_\mathsf{c}} & \sqrt{\mathsf{S_c}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \mathsf{M_c} \end{bmatrix} \begin{bmatrix} \mathsf{X_p} \\ \mathsf{X_c} \end{bmatrix} + \begin{bmatrix} \mathsf{Z_f} \\ \mathsf{Z_p} \\ \mathsf{Z_c} \end{bmatrix}, \tag{1}$$

where: the channel inputs are subject, without loss of generality, to the average power constraint $\mathbb{E}\left[|\mathsf{X}_i|^2\right] \leq 1$, $i \in \{\mathsf{p}, \mathsf{c}\}$ (i.e., non-unitary power constrains can be incorporated into the channel gains), $\mathsf{M_c}$ is a binary valued random-variable representing the state of the CTx, a $\star$ in the channel transfer matrix indicates the channel gain that does not affect the capacity region (because CTx can remove $\mathsf{X_c}$ from $\mathsf{Y_f}$), the channel parameters $(\mathsf{C}, \mathsf{S_p}, \mathsf{S_c}, \mathsf{I_p}, \mathsf{I_c}, \theta_\mathsf{p}, \theta_\mathsf{c}) \in \mathbb{R}_+^7$ are fixed and therefore known to all nodes, and the noises are independent proper-complex Gaussian random variables with, without loss of generality, zero-mean and unit variance. Some of the channel gains can be taken to be real-valued and non-negative because a node can compensate for the phase of one of its channel gains. In the following we assume that the channel sub-matrix $\begin{bmatrix} \sqrt{\mathsf{S_p}} & \sqrt{\mathsf{I_p}}\, e^{j\theta_\mathsf{p}} \\ \sqrt{\mathsf{I_c}}\, e^{j\theta_\mathsf{c}} & \sqrt{\mathsf{S_c}} \end{bmatrix}$ is full-rank (otherwise the channel is equivalent to a multiple access channel since the signals received at PRx and CRx are statistically equivalent [7]).

An often adopted figure of merit for the Gaussian channel is *gDoF* defined as follows. For some $\mathsf{SNR} > 0$ and for some non-negative $(\beta_\mathsf{f}, \beta_\mathsf{c}, \beta_\mathsf{p})$, consider the channel parameterization

$$\mathsf{S_c} = \mathsf{S_p} := \mathsf{SNR}^1, \;\; \mathsf{I_c} := \mathsf{SNR}^{\beta_\mathsf{c}}, \;\;\; \mathsf{I_p} := \mathsf{SNR}^{\beta_\mathsf{p}}, \;\;\; \mathsf{C} := \mathsf{SNR}^{\beta_\mathsf{f}}, \tag{2}$$

where β_i is the ratio of the received power on link $i \in \{\mathsf{p}, \mathsf{c}, \mathsf{f}\}$ expressed in dB over the received power on the direct link expressed in dB. The gDoF is defined as [3]

$$\mathsf{d} := \lim_{\mathsf{SNR}\to+\infty} \frac{\max\{\mathsf{R_p} + \mathsf{R_c}\}}{2\log(1+\mathsf{SNR})}, \tag{3}$$

where the maximization is intended over all possible achievable rates $(\mathsf{R_p}, \mathsf{R_c})$. The normalization

in (3) of the maximum sum-rate is with respect to the sum-capacity of an interference-free network. In the non-cooperative case $\mathsf{d}^{(\mathrm{NoCoop})} \leq 1$ because the absence of interference is the best possible scenario [3]. In the cooperative case, interference can be beneficial because it can carry useful cooperative information [9], [16] and, as a result, d in (3) can be larger than 1. In the limiting ideal case, where the CTx non-causally knows the PTx's message, the gDoF, indicated as $\mathsf{d}^{(\mathrm{Ideal})}$, can be unbounded [5]. In general, $\mathsf{d}^{(\mathrm{NoCoop})} \leq \mathsf{d} \leq \mathsf{d}^{(\mathrm{Ideal})}$.

The gDoF in (3) is an asymptotic exact characterization of the sum-capacity at high-SNR, in the sense that $\mathsf{R_p} + \mathsf{R_c} = 2\mathsf{d}\log(\mathsf{SNR}) + o(\log(\mathsf{SNR}))$ for $\mathsf{SNR} \gg 1$. At finite SNR the capacity is said to be known *to within* b *bits* if we can show an inner bound region $\mathcal{I}$ and an outer bound region $\mathcal{O}$ such that $(\mathsf{R_p}, \mathsf{R_c}) \in \mathrm{ConvexClosure}[\mathcal{I}] \Longrightarrow (\mathsf{R_p}+\mathsf{b}, \mathsf{R_c}+\mathsf{b}) \notin \mathcal{O}$ [3]. The gap is worst case capacity guarantee in the sense that inner and outer bounds are never too far away.

In this work we derive the gDoF and the constant gap for both the symmetric- and asymmetric-interference G-HD-CCIC. In particular: the interference-symmetric channel has $\beta_{\mathsf{p}} = \beta_{\mathsf{c}} = \alpha$, the Z-channel has $\beta_{\mathsf{p}} = \alpha$ and $\beta_{\mathsf{c}} = 0$, and the S-channel has $\beta_{\mathsf{c}} = \alpha$ and $\beta_{\mathsf{p}} = 0$, for some $\alpha \geq 0$. In the following, since no confusion arises, we shall simply set $\beta_{\mathsf{f}} = \beta$.

Before we proceed to the presentation of the results for the Gaussian noise case, we introduce the LDA model. The LDA is a noiseless high SNR approximation of (1) with input/output relationship $\mathsf{Y_f} = (1-\mathsf{M_c})\mathbf{S}^{n-n_{\mathsf{f}}}\mathsf{X_p}$, $\mathsf{Y_p} = \mathbf{S}^{n-n_{\mathsf{d}}}\mathsf{X_p} + \mathbf{S}^{n-n_{\mathsf{p}}}\mathsf{X_c}\ \mathsf{M_c}$ and $\mathsf{Y_c} = \mathbf{S}^{n-n_{\mathsf{c}}}\mathsf{X_p} + \mathbf{S}^{n-n_{\mathsf{d}}}\mathsf{X_c}\ \mathsf{M_c}$, where $(n_{\mathsf{d}}, n_{\mathsf{f}}, n_{\mathsf{p}}, n_{\mathsf{c}})$ are integers with $n := \max\{n_{\mathsf{f}}, n_{\mathsf{d}}, n_{\mathsf{p}}, n_{\mathsf{c}}\}$ such that $n_j/n_{\mathsf{d}} = \beta_j, j \in \{\mathsf{f}, \mathsf{p}, \mathsf{c}\}$, all vectors have length n and take value in GF(2), the sum is understood bit-wise on GF(2), and $\mathbf{S}$ is the down shift matrix of dimention n [20]. The model has the following interpretation. The PTx sends a length-n vector $\mathsf{X_p}$, whose top n_{d} bits are received at the PRx through the direct link, the top n_{c} bits are received at the CRx through the cross link, and the top n_{f} bits are received at the CTx through the cooperation link. Similarly, the CTx sends a length-n vector $\mathsf{X_c}$, whose top n_{d} bits are received at the CRx through the direct link and the top n_{p} bits are received at the PRx through the cross link. The fact that only a certain number of bits is observed at a given node is a consequence of the 'down shift' operation through $\mathbf{S}$. The bits not observed at a node are said to be 'below the noise floor' (since in the Gaussian

noise model these bits correspond to signals with a power less or equal than that of the noise). We shall use the LDA to inspire achievable schemes for the Gaussian noise channel in (1).

III. SUM-RATE OUTER BOUNDS

Here we specialize the known outer bounds for the bilateral FD source cooperation in [6], [7], [9] to the case of unilateral HD cooperation by following the approach of [18]. We can show

$$
\mathsf{R_p}+\mathsf{R_c} \le (\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{OB})} := \min\left\{ (\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{CS})}, (\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{DT})}, (\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{PV})} \right\} \quad \text{for} \tag{4}
$$

$$
\begin{aligned}
(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{CS})} &:= 2.507 + \min\Big\{\gamma \log\left(1+\mathsf{S}+\mathsf{C}\right) + 2\left(1-\gamma\right)\log\left(1+\mathsf{S}\right), \gamma\log\left(1+\mathsf{S}+\mathsf{I_c}\right) + \\
&+ \left(1-\gamma\right)\log\left(1+\left(\sqrt{\mathsf{S}}+\sqrt{\mathsf{I_c}}\right)^2+\left(\sqrt{\mathsf{S}}+\sqrt{\mathsf{I_p}}\right)^2+\left|\mathsf{S}+\sqrt{\mathsf{I_p}\mathsf{I_c}}\right|^2\right)\Big\}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{DT})} &:= \min\Bigg\{3+\gamma\log\left(1{+}\mathsf{S}\right)+\left(1-\gamma\right)\log\left(\frac{\max\left\{\mathsf{I_p},\mathsf{S}\right\}}{\mathsf{I_p}}\cdot\left(1{+}\left(\sqrt{\mathsf{S}}{+}\sqrt{\mathsf{I_p}}\right)^2\right)\right), \\
&\quad 2+\gamma\log\left(1+\mathsf{C}+\max\left\{\mathsf{I_c},\mathsf{S}\right\}\right)+\left(1-\gamma\right)\log\left(\frac{\max\left\{\mathsf{I_c},\mathsf{S}\right\}}{\mathsf{I_c}}\cdot\left(1+\left(\sqrt{\mathsf{S}}+\sqrt{\mathsf{I_c}}\right)^2\right)\right)\Bigg\}
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{PV})} &:= 5.503+\gamma\log\left(1+\mathsf{S}+\mathsf{C}+\mathsf{I_c}\right)+\left(1-\gamma\right)\log\left(1+\mathsf{I_p}+\frac{\mathsf{S}+2\sqrt{\mathsf{I_p}\mathsf{S}}}{1+\mathsf{I_c}}\right)+ \\
&+\left(1-\gamma\right)\log\left(1+\mathsf{I_c}+\frac{\mathsf{S}+2\sqrt{\mathsf{I_c}\mathsf{S}}}{1+\mathsf{I_p}}\right),
\end{aligned} \tag{7}
$$

by proceeding through the following steps: (i) in the outer bounds for the general memoryless FD channel with input $\mathsf{X_c}$, we substitute $\mathsf{X_c}$ with the pair $(\mathsf{X_c},\mathsf{M_c})$; (ii) for any triplet of random variables $(\mathsf{A},\mathsf{B},\mathsf{C})$ we bound $I(\mathsf{A},\mathsf{X_c},\mathsf{M_c};\mathsf{B}|\mathsf{C}) \le H(\mathsf{M_c}) + I(\mathsf{A},\mathsf{X_c};\mathsf{B}|\mathsf{C},\mathsf{M_c})$ since, for a binary-valued random variable $\mathsf{M_c}$, we have $I(\mathsf{M_c};\mathsf{B}|\mathsf{C}) \le H(\mathsf{M_c}) = \mathcal{H}(\gamma)$ for some $\gamma := \mathbb{P}[\mathsf{M_c}=0] \in [0,1]$ and where $\mathcal{H}(\gamma)$ is the binary entropy defined as $\mathcal{H}(\gamma) = -\gamma\log(\gamma) - (1-\gamma)\log(1-\gamma)$; (iii) for all the remaining mutual information terms, which are conditioned on $\mathsf{M_c}=\ell$, $\ell\in\{0,1\}$, the "Gaussian maximizes entropy"-principle guarantees that in order to exhaust all possible input distributions it suffices to consider joint Gaussian proper-complex inputs with covariance matrix $\begin{bmatrix} P_{\mathsf{p},\ell} & \rho_\ell\,\sqrt{P_{\mathsf{p},\ell}P_{\mathsf{p},\ell}} \\ \rho_\ell^*\,\sqrt{P_{\mathsf{p},\ell}P_{\mathsf{c},\ell}} & P_{\mathsf{c},\ell} \end{bmatrix}$ for $|\rho_\ell| \le 1$ and $(P_{\mathsf{p},0},P_{\mathsf{p},1},P_{\mathsf{c},0},P_{\mathsf{c},1}) \in \mathbb{R}_+^4$ satisfying the power constraint $\gamma P_{u,0} + \left(1-\gamma\right)P_{u,1} \le 1,\ u\in\{\mathsf{p},\mathsf{c}\}$; (iv) since PTx always transmits we define, for some $\delta\in[0,1]$, $P_{\mathsf{p},0} = \frac{\delta}{\gamma}$ and $P_{\mathsf{p},1} = \frac{1-\delta}{1-\gamma}$, while, since the CTx transmission only affects

the receivers outputs when $\mathsf{M_c} = 1$, we let $P_{\mathsf{c},0} = 0$ and $P_{\mathsf{c},1} = \frac{1}{1-\gamma}$; (v) the sum-rate upper bounds in (5) from [6, $\min\{$eq.(81)+eq.(82), eq.(83)$\}$], in (6) from [7, $\min\{$eq.(4d), eq.(4e)$\}$], and in (7) from [9, eq.(6)-(7)], are obtained by upper bounding each mutual information term over $(\rho_0, \rho_1, \delta) \in [0,1]^3$ while keeping γ fixed; (vi) each outer bound is also characterized by a linear combination of terms of the type "$\gamma \log(\gamma)$", which can be maximized over $\gamma \in [0,1]$ thus giving the additive constants in (5), (6) and (7). We do not report the steps of the derivation here because they are very similar to those in [17].

IV. The interference-symmetric G-HD-CCIC

When the interfering links have the same strength $\mathsf{I_c} = \mathsf{I_p}$, we obtain the interference-symmetric G-HD-CCIC, whose gDoF is

Theorem 1. *The gDoF of the interference-symmetric G-HD-CCIC is the solution of*

$$\mathsf{d}^{(\mathrm{SYM})} = \max_{\gamma \in [0,1]} \frac{1}{2} \min \Big\{ \gamma \max\{1, \beta\} + 2(1-\gamma), \tag{8a}$$

$$\gamma + (1-\gamma)\left(\max\{1,\alpha\} + [1-\alpha]^+\right), \tag{8b}$$

$$\gamma \max\{\alpha, \beta, 1\} + 2(1-\gamma)\max\{\alpha, 1-\alpha\} \Big\} \tag{8c}$$

$$= \begin{cases} 1-\alpha+\frac{1}{2}\frac{[\beta-2+2\alpha]^+ \ \alpha}{\beta+\alpha-1} & \alpha \in [0,1/2) \\ \alpha+\frac{1}{2}\frac{[\beta-2\alpha]^+ \ (2-3\alpha)}{\beta-3\alpha+1} & \alpha \in [1/2,2/3) \\ \max\left\{1-\frac{1}{2}\alpha, \frac{1}{2}\alpha\right\} & \alpha \in [2/3,2) \\ 1+\frac{1}{2}\frac{[\beta-2]^+ \ (\alpha-2)}{\beta+\alpha-3} & \alpha \in [2,\infty) \end{cases}. \tag{8d}$$

Proof: The converse follows by substituting the outer bound in (4) into (3) with the parameterization in (2) (with $\beta_{\mathsf{p}} = \beta_{\mathsf{c}} = \alpha$ and $\beta_{\mathsf{f}} = \beta$) and computing the limit. The achievability will follow from the constant gap result in Theorem 2. ■

The gDoF expression in (8d) has an interesting interpretation. Note $\mathsf{d}^{(\mathrm{NoCoop})} = \min\{1, \max\{1-\alpha, \alpha\}, \max\{1-\alpha/2, \alpha/2\}\}$ [3] and $\mathsf{d}^{(\mathrm{Ideal})} = \max\{1-\alpha/2, \alpha/2\}$ [5] for this topology.

- **Very weak interference:** For $\alpha \in [0, 2/3)$ and without cooperation $\beta = 0$ the tightest bound is (8c) [3]. Recall: no-cooperation in equivalent to $\gamma = 0$. For a $\beta > 0$, the bound in (8c) is optimized by $\gamma = 0$ whenever $\max\{1, \alpha, \beta\} \leq 2\max\{1-\alpha, \alpha\} = 2\mathsf{d}^{(\mathrm{NoCoop})}$, which is

equivalent to $\beta \leq 2\mathsf{d}^{(\text{NoCoop})}$. In other words, $\mathsf{d} > \mathsf{d}^{(\text{NoCoop})}$ only when β is larger than the sum-gDoF without cooperation. The optimal γ (fraction of time the CTx listens to the channel) is obtained by equating the bounds in (8c) and (8b) and given by

$$\gamma^{\star} = \frac{\min\{2-3\alpha, \alpha\}}{\min\{2-3\alpha, \alpha\} + \beta - 1}. \tag{9}$$

To see why the optimal γ is given by (9), let us analyze the LDA with deterministic switch. For the case $2 - 3\alpha \leq \alpha$, which corresponds to $\alpha \in [1/2, 2/3)$, an achievable scheme for the LDA is represented in Fig. 2(a) and Fig. 2(c).[1] In Phase 1 / Fig. 2(a), CTx listens to the channel and PTx sends the vector $[b_1[1], b_2]$, where $b_1[1]$ has normalized length 1 and b_2 has normalized length $\beta - 1$. Hence, over a fraction γ of the transmission time, CTx is aware of $\gamma(\beta-1)$ b_2-bits that PRx has not yet received. CTx will assist PTx to deliver these b_2-bits to PRx in Phase 2 in either of the following two cooperation modes.

Cooperation mode 1: CTx relays the b_2-bits to PRx on behalf of PTx in phase 2 by spending some of its own power to send these bits; these bits are also decoded at CRx, i.e., they are "common information" in the IC jargon [21], as shown in Figs. 2(d); decoding the interfering signal at CRx decreases the level of interference.

Cooperation mode 2: PTx re-sends the b_2-bits to PRx in phase 2; since these bits were known to CTx before transmission, CTx can treat them as "a state non-causally known at the transmitter but unknown at the receiver" and precode its transmit signal by using DPC [14]; the b_2-bits are not decoded at CRx, i.e., they are "private information" in the IC jargon [21], as shown in Figs. 2(c), 2(b) and 3; the net result of using DPC is that the achievable rate at CRx is as if the interference due to the b_2-bits was completely removed [14].

In Phase 2 / Fig. 2(c), CTx sends the vector $[b_{3c}, 0, b_{3p}{+}b_2]$, whose components have normalized lengths $2\alpha - 1$, $1 - \alpha$ and $1 - \alpha$, respectively. In the LDA, the linear combination $b_{3p} + b_2$ can be thought of as pre-coding the signal b_{3p} against the interference caused by b_2. PTx sends the

[1]In each sub-figure of Figs. 2 and 3, on the right hand side we represent the transmit signals X_p and X_c as vectors of normalized length $n/n_\mathsf{d} = \max\{1, \alpha, \beta\}$, and on the left hand side the received signals Y_p and Y_c as the sum of a certain down shifted version of the transmit signals. After the down-shift operation, the top part of a vector would be populated by zero; we do not represent these zeros and instead leave empty space in order not to clutter the figure. Note that the bits received at the same level at a node must be summed modulo-two.

vector $[b_{1c}, b_2, 0, b_{1p}]$, whose components have normalized lengths $2\alpha - 1$, $2 - 3\alpha$, $2\alpha - 1$ and $1 - \alpha$ (with an abuse of notation, here b_2 indicates the bits that have been received in Phase 1), respectively. CRx successively decodes b_{3c}, b_{1c}, b_{3p} in this order, while PRx successively decodes $b_{1c}, b_2, b_{3c}, b_{1p}$ in this order. Notice that CRx does not experience interference from b_2 when decoding b_{3p} (recall that on GF(2) $2b = 0$ for both $b = 0$ and $b = 1$). The achievable rates are $\mathsf{R_p} = \gamma \cdot 1 + (1 - \gamma) \cdot (2 - 2\alpha)$ and $\mathsf{R_c} = \gamma \cdot 0 + (1 - \gamma) \cdot \alpha$, thus giving the sum-rate $(\mathsf{R_p} + \mathsf{R_c})^{(\mathrm{IB})} = \gamma \cdot 1 + (1 - \gamma) \cdot (2 - \alpha)$. This sum-rate is larger than that without cooperation, given by $2\mathsf{d}^{(\mathrm{NoCoop})} = 2\alpha$ [3], if $\gamma \leq \frac{2-3\alpha}{1-\alpha}$. Next, $\gamma^\star$ in (9) is smaller than $\frac{2-3\alpha}{1-\alpha}$ only if $\beta > 2\alpha$. Thus, when $\beta \leq 2\alpha$, it would take too much time for the CTx to learn the message of the PTx and it is therefore better to not cooperate at all. This last observation gives an intuitive interpretation of why the gDoF in (8d) contains the term $[\beta - 2\alpha]^+$ for $\alpha \in [1/2, 2/3)$, i.e., because the gDoF without cooperation is improved by unilateral HD cooperation only when $\beta > 2\mathsf{d}^{(\mathrm{NoCoop})} = 2\alpha$. A similar reasoning may be done for the case $2 - 3\alpha > \alpha$, which corresponds to $\alpha \in [0, 1/2)$, for which an achievable scheme is given in Figs. 2(a) and 2(b). In this regime the gDoF without cooperation is improved by unilateral HD cooperation only when $\beta > 2\mathsf{d}^{(\mathrm{NoCoop})} = 2 - 2\alpha$.

• **Very strong interference:** For $\alpha \geq 2$ and without cooperation $\beta = 0$, the tightest bound is (8a) [3]. For a general $\beta > 0$, the bound in (8a) is optimized by $\gamma = 0$, which is equivalent to no-cooperation, whenever $\max\{1, \beta\} \leq 2 = 2\mathsf{d}^{(\mathrm{NoCoop})}$, which is equivalent to $\beta \leq 2\mathsf{d}^{(\mathrm{NoCoop})}$. Again we see that unilateral HD cooperation is beneficial in terms of gDoF only when β is larger than the sum-gDoF without cooperation. In this case the optimal γ is obtained by equating the bounds in (8a) and (8b) and given by

$$\gamma^\star = \frac{\alpha - 2}{\beta + \alpha - 3}. \tag{10}$$

To see why the optimal γ is given by (10), we again analyze the LDA with deterministic switch. Phase 1 is the same as in Fig. 2(a). In Phase 2 / Fig. 2(d), CTx sends $[b_3, b_2, 0]$, whose components have normalized lengths 1, $\alpha - 2$, and 1 (here b_2 indicates the bits that have been received in Phase 1), respectively. PTx sends $[b_1[2], 0]$, whose components have normalized lengths 1 and $\alpha - 1$, respectively. CRx successively decodes $b_1[2], b_3$ in this order. PRx successively decodes

$b_3, b_2, b_1[2]$ in this order. The achievable rates are $\mathsf{R_p} = \gamma \cdot 1 + (1-\gamma)\cdot(\alpha-1)$ and $\mathsf{R_c} = \gamma \cdot 0 + (1-\gamma)\cdot 1$, giving a sum-rate of $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = \gamma \cdot 1 + (1-\gamma)\cdot\alpha$. This sum-rate is larger than that without cooperation, given by $2\mathsf{d}^{(\mathrm{NoCoop})} = 2$ [3], if $\gamma \leq \frac{\alpha-2}{\alpha-1}$. Next, $\gamma^\star$ in (10) is smaller than $\frac{\alpha-2}{\alpha-1}$ only if $\beta > 2$. Again, the interpretation is that if $\beta \leq 2$ it takes too long to transfer bits from PTx to CTx and hence it is preferable not to cooperate. This last observation gives an intuitive interpretation of why the gDoF in (8d) contains the term $[\beta-2]^+$ for $\alpha \in [2,\infty)$, i.e., because the gDoF without cooperation is improved only when $\beta > 2\mathsf{d}^{(\mathrm{NoCoop})} = 2$.

• **Moderately weak and strong interference:** For $\alpha \in [2/3, 2)$ and without cooperation $\beta = 0$ the bound in (8b) is the tightest [8]. The bound in (8b) is always optimized by $\gamma = 0$, which is equivalent to the case of no-cooperation. Hence, in this regime it is always gDoF-optimal to operate the channel as a non-cooperative IC no matter how strong the cooperation link is, or in other words, unilateral HD cooperation does not help in managing interference.

Now we show that the upper bound in (4) is achievable to within a constant gap. This will imply the achievability of the gDoF upper bound in Theorem 1. We have

Theorem 2. *The sum-capacity upper bound in* (4) *is achievable to within 10.503 bits regardless of the actual value of the channel parameters for the interference-symmetric G-HD-CCIC.*

Proof: The proof can be found in Appendix A, where we develop achievable schemes inspired by the LDA strategies in Fig. 2. ■

Fig. 4(a) shows the optimal gDoF and the gap for the interference-symmetric G-HD-CCIC. The whole set of parameters has been partitioned into multiple sub-regions depending upon different levels of cooperation (β) and interference (α) strengths. These regimes are numbered from 1 to 10 and the details appear in Appendix A. We end this analysis with few comments:

1) Everywhere, except in regions 3, 8 and 10 in Fig. 4(a), unilateral cooperation might not be worth implementing since the same gDoF is achieved without cooperation.

2) The symmetric G-HD-CCIC has the same gDoF of the non-causal cognitive channel in regions 4 and 5 in Fig. 4(a). Thus, in these two regions, the performance of the system, in terms of gDoF, is not worsened by allowing causal learning at CTx.

3) In regions 1, 4, 5 and 6 of Fig. 4(a) the gDoF equals that of the equivalent FD channel [17] and is equal to the non-cooperative case. Since the FD channel is an outer bound to the HD channel and no-cooperative strategies are possible under the HD constraint, we conclude that in these regimes the same gap results found for the FD case [17] hold in the case of HD source cooperation. In this case, gDoF-wise, there is no loss in having a HD CTx compared to a more powerful FD CTx.
4) All the achievable schemes use successive decoding at the nodes, which, in practice, is simpler than joint decoding. Thus our proposed schemes, which are optimal to within a constant gap, may be used as guidelines to deploy practical cognitive radio systems.
5) The gap computed in this work is around 5 bits bigger than that computed in the corresponding FD case [17]. Possible ways to reduce the gap may be: (i) apply joint decoding at the receivers; (ii) develop block Markov schemes instead of taking inspiration by the LDA; (iii) develop achievable strategies that exploit the randomness into the switch to convey further useful information; (iv) develop tighter upper bounds than those used in this work, especiallyfor $\alpha \leq \frac{2}{3}$ where the gap is quite large.

V. THE INTERFERENCE-ASYMMETRIC G-HD-CCIC

In this section we analyze two interference-asymmetric G-HD-CCIC models, the Z- and S-channels. These two cases are analyzed separately because of the asymmetry in the cooperation.

A. The Z-channel

With $\mathsf{I_c} = 0$ we obtain the Z-channel, whose gDoF is

Theorem 3. *The gDoF of the Z-channel is*

$$\mathsf{d}^{(\mathrm{Z})} = \frac{1}{2} \max_{\gamma \in [0,1]} \min \left\{ \gamma \max\left\{1, \beta\right\} + (1-\gamma) 2,\ \gamma + (1-\gamma) \left(\max\left\{1, \alpha\right\} + [1-\alpha]^+ \right) \right\}$$

given by

$$\mathsf{d}^{(\mathrm{Z})} = \begin{cases} \max\left\{1 - \frac{1}{2}\alpha, \frac{1}{2}\alpha\right\} & \alpha \in [0,2) \\ 1 + \frac{1}{2} \frac{[\beta-2]^+ \ (\alpha-2)}{\beta+\alpha-3} & \alpha \in [2, \infty) \end{cases} . \tag{11}$$

Proof: The converse follows by substituting the outer bound in (4) into (3) with the parameterization in (2) (with $\beta_{\mathsf{c}} = 0$, $\beta_{\mathsf{p}} = \alpha$ and $\beta_{\mathsf{f}} = \beta$) and computing the limit. The achievability will follow from the constant gap result in Theorem 4. ∎

For future reference, $\mathsf{d}^{(\mathrm{NoCoop})} = \min\{1, \max\{1 - \alpha/2, \alpha/2\}\}$ [3] and $\mathsf{d}^{(\mathrm{Ideal})} = \max\{1 - \alpha/2, \alpha/2\}$ [5] for this topology, hence cooperation can improve performance only in very strong interference, i.e., $\alpha > 2$. The interpretation of the gDoF in (11) is similar to that of the interference-symmetric case in (8d). In particular, if the channel has weak or strong interference, i.e., $\alpha \leq 2$, the gDoF is the same as for the non-cooperative Z-channel [22]; in this regime it might not be worth to engage in unilateral cooperation. In very strong interference, i.e., $\alpha > 2$, unilateral cooperation gives larger gDoF than in the case of no-cooperation only when $\beta > 2\mathsf{d}^{(\mathrm{NoCoop})} = 2$. An achievable scheme for the LDA in this regime is exactly the same developed for the corresponding interference-symmetric channel in Figs. 2(a) and 2(d), with the only difference that now the signal $\mathsf{X}_{\mathsf{p}}[2]$ is not received at $\mathsf{Y}_{\mathsf{c}}[2]$ since $\beta_{\mathsf{c}} = 0$.

Now we show that the upper bound in (4) is achievable to within a constant gap. This will imply the achievability of the gDoF upper bound in Theorem 3. We have

Theorem 4. *The sum-capacity upper bound in* (4) *is achievable to within 4.507 bits regardless of the actual value of the channel parameters for the Z-channel.*

Proof: The proof can be found in Appendix B, where we develop achievable schemes inspired by the LDA strategies in Figs. 2(a) and 2(d). ∎

Fig. 4(b) shows the optimal gDoF and the gap for the Z-channel. The whole set of parameters has been partitioned into multiple sub-regions depending upon different levels of cooperation (β) and interference (α) strengths. These regimes are numbered from 1 to 5 and the details for the gap computation appear in Appendix B. We conclude with few comments:

1) We notice that in regions 1, 4 and 5 of Fig. 4(b) the gDoF of the HD channel is as that in FD [17]. Thus, the same gap results found for the FD case hold in the HD case. Moreover, in region 2 in Fig. 4(b) the gDoF equals that of the classical IC. Hence in regions 1, 2, 4 and 5 cooperation might not be worth implementing since the same gDoF is attained without cooperation.

2) The Z-channel achieves the same gDoF of the non-causal cognitive channel everywhere except in $\alpha > 2$ (regions 1, 2 and 3 in Fig. 4(b)). Therefore, for $\alpha \leq 2$, causal cognition attains the ultimate performance of the ideal non-causal cognitive radio Z-channel.

3) By comparing Figs. 4(a) and 4(b), we observe that the gDoF of the Z-channel is always greater than or equal to that of the interference-symmetric G-HD-CCIC. This is due to the fact that the PTx does not cooperate in sending the signal of CTx. Therefore by removing the link between PTx and CRx we rid CRx of only interfering signals. We observe that the Z-channel outperforms the symmetric channel when $0 \leq \alpha \leq \frac{2}{3}$.

B. The S-channel

With $\mathsf{I_p} = 0$ we obtain the S-channel, whose gDoF is

Theorem 5. *The gDoF of the S-channel is*

$$\begin{aligned}
\mathsf{d}^{(\mathrm{S})} &= \frac{1}{2} \max_{\gamma \in [0,1]} \min \left\{ \gamma + 2\,(1-\gamma)\,, \gamma \max\left\{\beta, \alpha, 1\right\} + (1-\gamma)\left(\max\left\{1, \alpha\right\} + [1-\alpha]^+\right) \right\} \\
&= \begin{cases} 1 - \frac{1}{2}\alpha + \frac{1}{2}\frac{\alpha[\alpha+\beta-2]^+}{\beta+\alpha-1} & \alpha \in [0,1) \\ \frac{1}{2}\alpha + \frac{1}{2}\frac{(2-\alpha)[\beta-\alpha]^+}{\beta-\alpha+1} & \alpha \in [1,2) \\ 1 & \alpha \in [2,\infty) \end{cases} .
\end{aligned} \tag{12}$$

Proof: The converse follows by substituting the outer bound in (4) into (3) with the parameterization in (2) (with $\beta_{\mathsf{p}} = 0$, $\beta_{\mathsf{c}} = \alpha$ and $\beta_{\mathsf{f}} = \beta$) and computing the limit. The achievability will follow from the constant gap result in Theorem 6. ■

For future reference, $\mathsf{d}^{(\mathrm{NoCoop})} = \min\{1, \max\{1-\alpha/2, \alpha/2\}\}$ [3] and $\mathsf{d}^{(\mathrm{Ideal})} = 1$ [5] for this topology, hence cooperation can improve performance only when the channel is not in very strong interference, i.e., $\alpha < 2$. Also in this case the interpretation of the gDoF in (12) is similar to that of the interference-symmetric case in (8d). In particular, if the channel has very strong interference, i.e., $\alpha > 2$, the gDoF is the same as for the non-cooperative S-channel [22]; in this regime it might not be worth to engage in unilateral cooperation. In weak and strong interference, i.e., $\alpha \leq 2$, unilateral cooperation gives larger gDoF than in the case of no-cooperation only when $\beta > 2\mathsf{d}^{(\mathrm{NoCoop})} = 2\max\{1-\alpha/2, \alpha/2\}$. A representation of the LDA

schemes used for $\alpha < 2$ and $\beta > 2\max\{1-\alpha/2, \alpha/2\}$ is given in Figs. 2(a), 3(a) and 3(b), which can be interpreted as done for the interference-symmetric case in Fig. 2.

Now we show that the upper bound in (4) is achievable to within a constant gap. This will imply the achievability of the gDoF upper bound in Theorem 5. We have

Theorem 6. *The sum-capacity upper bound in* (4) *is achievable to within 5 bits regardless of the actual value of the channel parameters for the S-channel.*

Proof: The proof can be found in Appendix C, where we develop achievable schemes inspired by the LDA strategies in Figs. 2(a), 3(a) and 3(b). ■

Fig. 4(c) shows the optimal gDoF and the gap for the S-channel. The whole set of parameters has been partitioned into multiple sub-regions depending upon different levels of cooperation (β) and interference (α) strengths. The details for the gap computation appear in Appendix C. We conclude the analysis of the S-channel with few comments:

1) There are some regions (1 and 2 in Fig. 4(c)) in which the gDoF of the HD channel is as that in FD. In these regions, the same additive gap results found for the FD case [17] hold in HD. Moreover, in region 3 in Fig. 4(c) the gDoF equals that of the pure IC.

2) The S-channel achieves the same gDoF of the non-causal cognitive channel, i.e., $\mathsf{d} = 1$, for $\alpha \geq 2$ (region 1 in Fig. 4(c)). Thus, in this region the S-channel attains the ultimate performance of the ideal non-causal cognitive radio S-channel .

3) The S-channel outperforms the interference-symmetric G-HD-CCIC when either $0 \leq \alpha \leq \frac{2}{3}$ or when $\alpha \leq 2$ and $\beta \geq \max\{2-\alpha, \alpha\}$ (regions 4 and 5, and parts of regions 2 and 3 in Fig. 4(c)). On the other hand, the interference-symmetric G-HD-CCIC outperforms the S-channel in very strong interference and strong cooperation, i.e., $\min\{\alpha, \beta\} \geq 2$. This is so because, in the very strong interference and cooperation regime, the system performance is enhanced by allowing the CTx to help the PTx to convey the information, but this is not possible since $\mathsf{I}_\mathsf{p} = 0$.

4) When $\alpha \geq 2$ (region 1 in Fig. 4(c)) we have an exact sum-capacity result, i.e., the gap between the sum-rate outer bound and inner bound is equal to zero.

VI. CONCLUSIONS

In this work we studied the Gaussian causal cognitive interference channel where the cognitive source, who helps the primary source, is constrained to operate in half-duplex mode. From an application standpoint, this model fits future 4G cellular networks with heterogeneous deployments. We analyzed both the interference-symmetric and interference-asymmetric channels, which correspond to different network deployments. We determined, for each topology, the gDoF and the sum-capacity to within a constant gap. The proposed schemes are relatively simple in the sense that use successive decoding rather than joint decoding. We compared the interference-symmetric and interference-asymmetric models by highlighting the regimes where the gDoF is as that of the classical IC without cooperation, i.e., regimes where cooperation might not be worth implementing, and by identifying the regimes where the system attains the ultimate limits predicted by the ideal non-causal cognitive model, i.e., regimes where the performance is not worsened by allowing causal half-duplex learning at the cognitive source.

APPENDIX A

CONSTANT GAP RESULT FOR THE INTERFERENCE-SYMMETRIC G-HD-CCIC

Let $\mathsf{I}_\mathsf{p} = \mathsf{I}_\mathsf{c} = \mathsf{I}$ and $\mathsf{d}^{(\mathrm{SYM})} = \mathsf{d}$ for brevity. We analyze different regimes by developing simple achievable strategies. We define $\mathsf{GAP} = (\mathsf{R}_\mathsf{p} + \mathsf{R}_\mathsf{c})^{(\mathrm{OB})} - (\mathsf{R}_\mathsf{p} + \mathsf{R}_\mathsf{c})^{(\mathrm{IB})}$.

- **Regime 1: Very Strong Interference 1**: $\alpha \geq 2$, $\beta \leq 1$.

Parameter Range: $\mathsf{I} \geq \mathsf{S}(1+\mathsf{S})$ and $\mathsf{C} \leq \mathsf{S}$, in which case we have $\mathsf{d} \leq 1$ (with $\gamma = 0$). In this regime the gDoF upper bound coincides with that in FD [17]. Thus, as in FD, $\mathsf{GAP} \leq 1$ bit.

- **Regime 2: Very Strong Interference 2**: $\alpha \geq 2$ and $1 < \beta \leq 2$.

Parameter Range: $\mathsf{I} \geq \mathsf{S}(1+\mathsf{S})$ and $\mathsf{S} < \mathsf{C} \leq \mathsf{S}(\mathsf{S}+1)$, in which case $\mathsf{d} \leq 1$ (with $\gamma = 0$).

Inner Bound: classical IC in very strong interference, i.e., $(\mathsf{R}_\mathsf{p} + \mathsf{R}_\mathsf{c})^{(\mathrm{IB})} = 2\log(1+\mathsf{S})$, which implies $\mathsf{d} = 1$. This shows the achievability of the gDoF upper bound.

Outer Bound: in this regime the tightest sum-rate outer bound is given by (5), which can be further upper-bounded as $(\mathsf{R}_\mathsf{p} + \mathsf{R}_\mathsf{c})^{(\mathrm{OB})} \leq 2.507 + 2\log(1+\mathsf{S})$. Thus, $\mathsf{GAP} \leq 2.507$ bits.

- **Regime 3: Very Strong Interference 3**: $\alpha \geq 2$, $\beta > 2$.

Parameter Range: $\mathsf{I} \geq \mathsf{S}(1+\mathsf{S})$ and $\mathsf{C} > \mathsf{S}(\mathsf{S}+1)$, where we have $2\mathsf{d} \leq \max_\gamma \min\left\{\gamma\beta+2\left(1-\gamma\right), \gamma+\left(1-\gamma\right)\alpha\right\}$. In this expression the first term is increasing in γ while the second one is decreasing in γ. Thus the optimal γ can be found by equating the two terms and it is given by $\gamma^\star = \frac{\alpha-2}{\beta+\alpha-3}$ in (10), which leads to $\mathsf{d} \leq \frac{1}{2}\frac{\beta\alpha-2}{\beta+\alpha-3}$.

Inner Bound: We use the achievable scheme in Appendix D-B to show that, with $\gamma' = \frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x} \overset{\mathsf{SNR}\gg 1}{\to} \gamma^\star$ for $x := \log\left(1+\frac{\mathsf{I}}{(1+\mathsf{S})^2}\right) \overset{\mathsf{SNR}\gg 1}{\to} \alpha-2$ (recall that $\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right) \overset{\mathsf{SNR}\gg 1}{\to} \beta-1$), the following sum-rate is achievable:

$$\begin{aligned}(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} &= \gamma' \log\left(1+\mathsf{S}\right) - \gamma' \log\left(1+\frac{\mathsf{S}}{1+\mathsf{S}}\right) + \gamma' \log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right) \\ &+ \left(1-\gamma'\right)\log\left(1+\mathsf{S}\right) + \left(1-\gamma'\right)\log\left(1+\mathsf{S}\right) - \left(1-\gamma'\right)\log\left(1+\frac{\mathsf{S}}{1+\mathsf{S}}\right)\end{aligned} \tag{13}$$

implying $2\mathsf{d} = \frac{\alpha-2}{\beta+\alpha-3}[1-0+(\beta-1)] + \frac{\beta-1}{\beta+\alpha-3}[1+1-0] = \frac{\alpha\beta-2}{\alpha+\beta-3}$ (the gDoF upper bound is achievable).

Outer Bound: in this regime the tightest sum-rate outer bounds are those in (5) and (6). With the bound in (5), $\mathsf{GAP} \leq 3.507$ bits; with the bound in (6), $\mathsf{GAP} \leq 5$ bits. Thus, $\mathsf{GAP} \leq 5$ bits.

- **Regime 4: Strong Interference**: $1 \leq \alpha < 2$.

Parameter Range: $\mathsf{S} \leq \mathsf{I} < \mathsf{S}(1+\mathsf{S})$, in which case we have $\mathsf{d} \leq \frac{\alpha}{2}$ (with $\gamma = 0$). In this regime the gDoF upper bound coincides with that in FD [17]. Thus, as in FD, $\mathsf{GAP} \leq 1$ bit.

- **Regime 5: Moderately Weak Interference**: $\frac{2}{3} \leq \alpha < 1$.

Parameter Range: $\mathsf{S} < \mathsf{I}$, in which case we have $\mathsf{d} \leq 1 - \frac{\alpha}{2}$ (with $\gamma = 0$). In this regime the gDoF upper bound coincides with that in FD [17]. Thus, as in FD, $\mathsf{GAP} \leq 3$ bits.

- **Regime 6: Weak Interference 1**: $\frac{1}{2} \leq \alpha < \frac{2}{3}$ and $\beta \leq 2\alpha - 1$.

Parameter Range: $\mathsf{S}(\mathsf{S}+1) > \mathsf{I}(\mathsf{I}+1)^2$, $\mathsf{I}^2 \geq \mathsf{S}$ and $\mathsf{C} \leq \frac{\mathsf{I}^2}{\mathsf{S}}$, in which case we have $\mathsf{d} \leq \alpha$ (with $\gamma = 0$). The gDoF upper bound coincides with that in FD [17]. Thus, as in FD, $\mathsf{GAP} \leq 7.322$ bits.

- **Regime 7: Weak Interference 2**: $\frac{1}{2} \leq \alpha < \frac{2}{3}$ and $2\alpha - 1 < \beta \leq 2\alpha$.

Parameter Range: $\mathsf{S}(\mathsf{S}+1) > \mathsf{I}(\mathsf{I}+1)^2$, $\mathsf{I}^2 \geq \mathsf{S}$ and $\frac{\mathsf{I}^2}{\mathsf{S}} < \mathsf{C} \leq \mathsf{I}^2$, where we have $\mathsf{d} \leq \alpha$ (with $\gamma = 0$).

Inner Bound: classical IC with the power split of [3], i.e., $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = 2\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{\mathsf{I}}\right) - 2$, which implies $\mathsf{d} = \alpha$. Thus, the gDoF upper bound is achievable.

Outer Bound: in this regime the tightest sum-rate outer bound is given by (7), that can be further

upper bounded as $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{OB})} \leq 5.503 + 2\log\left(1+\mathsf{I}+\frac{\mathsf{S}+2\sqrt{\mathsf{IS}}}{\mathsf{I}+1}\right)$.

Gap: $\mathsf{GAP} \leq 7.503 + 2\log\left(1+\frac{1+\mathsf{I}+2\sqrt{\mathsf{IS}}}{\mathsf{I}+\mathsf{I}^2+\mathsf{S}}\right) + 2\log\left(\frac{\mathsf{I}}{1+\mathsf{I}}\right) \leq 9.503$ bits.

- **Regime 8: Weak Interference 3**: $\frac{1}{2} \leq \alpha < \frac{2}{3}$, $\beta > 2\alpha$.

Parameter Range: $\mathsf{S}(\mathsf{S}+1) > \mathsf{I}(\mathsf{I}+1)^2$, $\mathsf{I}^2 \geq \mathsf{S}$ and $\mathsf{C} > \mathsf{I}^2$, in which case we have $2\mathsf{d} \leq \max_\gamma \min\left\{\gamma + (1-\gamma)(2-\alpha), \gamma\beta + 2(1-\gamma)\alpha\right\}$. In this expression the first term is decreasing in γ while the second term is increasing in γ. Thus the optimal γ can be found by equating the two terms and it is given by $\gamma^\star = \frac{2-3\alpha}{\beta-3\alpha+1}$ in (9), which leads to $\mathsf{d} \leq \frac{1}{2}\frac{2\beta-\alpha\beta-2\alpha}{\beta-3\alpha+1}$.

Inner Bound: the following sum-rate is achievable (Appendix D-C):

$$\begin{aligned}(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} &= \gamma'\log(1+\mathsf{S}) - \gamma'\log\left(1+\frac{\mathsf{S}}{1+\mathsf{S}}\right) + \gamma'\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right) \\ &+ 2(1-\gamma')\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) - 2(1-\gamma')\log\left(1+\frac{\mathsf{I}}{1+\mathsf{I}}\right) \\ &+ (1-\gamma')\log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}+\frac{\mathsf{I}}{1+\mathsf{I}}\right) - (1-\gamma')\log\left(1+\frac{\mathsf{SI}+\mathsf{I}+\mathsf{I}^2}{(1+\mathsf{I})^2}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) \end{aligned} \tag{14}$$

where $\gamma' = \frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x} \overset{\mathsf{SNR}\gg 1}{\to} \gamma^\star$ for $x := \log\left(1+\frac{\mathsf{S}^2}{(1+\mathsf{I})^3+\mathsf{S}+\mathsf{SI}}\right) \overset{\mathsf{SNR}\gg 1}{\to} 2-3\alpha$. The sum-rate in (14) implies $2\mathsf{d} = \frac{2-3\alpha}{\beta-3\alpha+1}[1-0+(\beta-1)] + \frac{\beta-1}{\beta-3\alpha+1}[\alpha-0+\alpha-(1-\alpha)+(1-\alpha)-0] = \frac{2\beta-\alpha\beta-2\alpha}{\beta-3\alpha+1}$. Thus, the gDoF upper bound is achievable.

Outer Bound: in this regime the tightest sum-rate outer bounds are those in (6) and (7). With the bound in (6), $\mathsf{GAP} \leq 8$ bits; with the bound in (7), $\mathsf{GAP} \leq 10.503$ bits. Thus, $\mathsf{GAP} \leq 10.503$ bits.

- **Regime 9: Weak interference 4**: $\alpha < \frac{1}{2}$ $\beta \leq 2-2\alpha$.

Parameter Range: $\mathsf{I}(\mathsf{I}+1) < \mathsf{S}$ and $\mathsf{C} \leq \frac{\mathsf{S}^2}{\mathsf{I}^2}$, in which case we have $\mathsf{d} \leq 1-\alpha$ (with $\gamma = 0$).

Inner Bound: classical IC with the power split of [3], i.e., $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = 2\log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}\right)$, which implies $\mathsf{d} = 1-\alpha$. This shows the achievability of the gDoF upper bound.

Outer Bound: in this regime the tightest sum-rate outer bound is given by (7), that can be further upper bounded as $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{OB})} \leq 5.503 + 2\log\left(1+\mathsf{I}+\frac{\mathsf{S}+2\sqrt{\mathsf{IS}}}{\mathsf{I}+1}\right)$.

Gap: $\mathsf{GAP} \leq 5.503 + 2\log\left(1+\frac{\mathsf{I}^2+\mathsf{I}+2\sqrt{\mathsf{SI}}}{1+\mathsf{I}+\mathsf{S}}\right) \leq 5.503 + 2\log(1+2) = 8.673$ bits.

- **Regime 10: Weak Interference 5**: $\alpha < \frac{1}{2}$, $\beta > 2-2\alpha$.

Parameter Range: $\mathsf{I}(\mathsf{I}+1) < \mathsf{S}$ and $\mathsf{C} > \frac{\mathsf{S}^2}{\mathsf{I}^2}$, in which case we have

$2\mathsf{d} \leq \max_\gamma \min\left\{\gamma + (1-\gamma)(2-\alpha), \gamma\beta + 2(1-\gamma)(1-\alpha)\right\}$. In this expression the first term

is decreasing in γ while the second term is increasing in γ. Thus the optimal γ can be found by equating the two terms and it is given by $\gamma^\star = \frac{\alpha}{\beta+\alpha-1}$ in (9), which leads to $\mathsf{d} \leq \frac{1}{2}\frac{2\beta+2\alpha-\alpha\beta-2}{\beta+\alpha-1}$.

Inner Bound: the following sum-rate is achievable (Appendix D-D):

$$(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = \gamma' \log(1+\mathsf{S}) - \gamma' \log\left(1+\frac{\mathsf{S}}{1+\mathsf{S}}\right) + \gamma' \log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right) - (1-\gamma')\log(1+\mathsf{I})$$
$$+ (1-\gamma')\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) + (1-\gamma')\log\left(1+\frac{\mathsf{I}}{1+\mathsf{I}}+\mathsf{S}\right) - (1-\gamma')\log\left(1+\frac{\mathsf{I}}{1+\mathsf{I}}\right) \quad (15)$$

where $\gamma' = \frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x} \overset{\mathsf{SNR}\gg 1}{\to} \gamma^\star$ for $x := \log\left(1+\frac{\mathsf{SI}}{(1+\mathsf{I})^2+\mathsf{S}}\right) \overset{\mathsf{SNR}\gg 1}{\to} \alpha$. The sum-rate in (15) implies $2\mathsf{d} = \frac{\alpha}{\beta+\alpha-1}[1-0+(\beta-1)] + \frac{\beta-1}{\beta+\alpha-1}[(1-\alpha)-\alpha+1-0] = \frac{2\beta+2\alpha-\alpha\beta-2}{\beta+\alpha-1}$. Thus, the gDoF upper bound is achievable.

Outer Bound: in this regime the tightest sum-rate outer bounds are those in (6) and (7). With the bound in (6), $\mathsf{GAP} \leq 6$ bits; with the bound in (7), $\mathsf{GAP} \leq 9.503$ bits. Thus, $\mathsf{GAP} \leq 9.503$ bits.

APPENDIX B
CONSTANT GAP RESULT FOR THE Z-CHANNEL

Let $\mathsf{I_p} = \mathsf{I}$, $\mathsf{I_c} = 0$ and $\mathsf{d}^{(\mathrm{Z})} = \mathsf{d}$ for brevity. We consider two different regimes.

• **Regime 1: Strong and Very Strong Interference:** $\alpha \geq 1$.

The analysis is similar to that of the symmetric G-HD-CCIC in the same regime (we use the same achievable strategies by setting $\mathsf{I_c} = 0$ both in the inner and outer bounds) and the gap is at most 4.507 bits. In particular in regions 1 and 4 in Fig. 4(b), the gDoF is equal to that in FD [17]. Since, with no cooperation, FD represents an outer bound for HD, in these two regimes the gaps are as those in [17] ($\mathsf{GAP} \leq 1$ bit).

• **Regime 2: Weak Interference:** $\alpha < 1$.

Parameter Range: $\mathsf{I} < \mathsf{S}$, in which case we have (with $\gamma = 0$) $\mathsf{d} \leq 1 - \frac{\alpha}{2}$.

Inner Bound: classical non-cooperative Z-channel in weak interference [22, Theorem 2], i.e., $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = \log(1+\mathsf{S}) + \log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}\right)$, which implies $\mathsf{d} = 1-\frac{1}{2}\alpha$ (the gDoF upper bound is achievable). The gDoF upper bound coincides with that in FD [17], so $\mathsf{GAP} \leq 1$ bit.

APPENDIX C

CONSTANT GAP RESULT FOR THE S-CHANNEL

Let $\mathsf{I_c}=\mathsf{I}$, $\mathsf{I_p}=0$ and $\mathsf{d}^{(\mathrm{S})}=\mathsf{d}$ for brevity. We analyze different regimes.

• **Regime 1:** $\alpha\geq 2$. The gDoF is as for the non-cooperative S-channel. In this regime the gDoF upper bound coincides with that in FD [17], so $\mathsf{GAP}=0\,\mathrm{bit}$.

• **Regime 2:** $\alpha<2$ **and** $\beta\leq\max\{1,\alpha\}$. In this regime the gDoF is as for the non-cooperative S-channel. The gDoF upper bound coincides with that in FD [17]. Thus, as in FD, $\mathsf{GAP}=2$ bits.

• **Regime 3 :** $\alpha\leq 1$ **and** $1<\beta\leq 2-\alpha$**.**

Parameter Range: $\mathsf{I}\leq\mathsf{S}$ and $\mathsf{S}<\mathsf{C}\leq\frac{\mathsf{S}^2}{\mathsf{I}}$, in which case we have (with $\gamma=0$) $\mathsf{d}=1-\frac{\alpha}{2}$.

Inner Bound: classical non-cooperative S-channel in weak interference [22, Theorem 2], i.e., $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})}=\log\left(1+\mathsf{S}\right)+\log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}\right)$, which implies $\mathsf{d}=1-\frac{1}{2}\alpha$. This shows that the gDoF upper bound is achievable.

Outer Bound: in this regime the tightest sum-rate outer bound is given by (6), that can be further upper bounded as $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{OB})}\leq 2+\log\left(1+\left(\sqrt{\mathsf{I}}+\sqrt{\mathsf{S}}\right)^2\right)+\log\left(\frac{\mathsf{S}}{\mathsf{I}}\right)$.

Gap: $\mathsf{GAP}\leq 2+\log\left(1+\frac{2\sqrt{\mathsf{SI}}}{1+\mathsf{I}+\mathsf{S}}\right)+\log\left(1+\frac{1}{\mathsf{I}}\right)\leq 4$ bits.

• **Regime 4**: $1\leq\alpha<2$ and $\beta>\alpha$.

Parameter Range: $\mathsf{S}\leq\mathsf{I}<\mathsf{S}(\mathsf{S}+1)$ and $\mathsf{C}>\mathsf{I}$, in which case we have

$2\mathsf{d}\leq\max_\gamma\min\left\{\gamma+2\left(1-\gamma\right),\gamma\beta+\left(1-\gamma\right)\alpha\right\}$. In this expression the first term is decreasing in γ while the second term is increasing in γ. Thus the optimal γ can be found by equating the two terms and it is given by $\gamma^\star=\frac{2-\alpha}{\beta-\alpha+1}$, which leads to $\mathsf{d}\leq\frac{1}{2}\frac{2\beta-\alpha}{\beta-\alpha+1}$.

Inner Bound: inspired by the LDA scheme in Figs. 2(a) and 3(b), it is possible to show that the following rate, with $\gamma'=\frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x}\overset{\mathsf{SNR}\gg 1}{\rightarrow}\gamma^\star$ for $x:=\log\left(1+\frac{\mathsf{S}^2}{1+\mathsf{I}}\right)\overset{\mathsf{SNR}\gg 1}{\rightarrow}2-\alpha$, is achievable

$$\begin{aligned}(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})}&=\log\left(1+\mathsf{S}\right)-\gamma'\log\left(1+\frac{\mathsf{S}}{1+\mathsf{S}}\right)+\gamma'\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)\\&+\left(1-\gamma'\right)\log\left(1+\mathsf{S}+\mathsf{I}\right)-\left(1-\gamma'\right)\log\left(1+\frac{\mathsf{SI}}{1+\mathsf{I}}+\mathsf{S}\right),\end{aligned}\tag{16}$$

The sum-rate in (16) implies $2\mathsf{d}=1+\frac{2-\alpha}{\beta-\alpha+1}[-0+(\beta-1)]+\frac{\beta-1}{\beta-\alpha+1}[\alpha-1]=\frac{2\beta-\alpha}{\beta-\alpha+1}$. Thus, the

gDoF upper bound is achievable.

Outer Bound: the tightest sum-rate outer bound is that in (6) (with $\mathsf{I_p} = 0$). With the first constraint in (6), $\mathsf{GAP} \leq 4$ bits; with the second one in (6), $\mathsf{GAP} \leq 4$ bits. Thus, $\mathsf{GAP} \leq 4$ bits.

• **Regime 5:** $\alpha < 1$ **and** $\beta > 2 - \alpha$.

Parameter Range: $\mathsf{I} < \mathsf{S}$ and $\mathsf{C} > \frac{\mathsf{S}^2}{\mathsf{I}}$, in which case we have
$2\mathsf{d} \leq \max_\gamma \min\left\{\gamma + 2\left(1-\gamma\right), \gamma\beta + \left(1-\gamma\right)\left(2-\alpha\right)\right\}$. In this expression the first term is decreasing in γ while the second term is increasing in γ. Thus the optimal γ can be found by equating the two terms and is given by $\gamma^\star = \frac{\alpha}{\beta+\alpha-1}$, which leads to $\mathsf{d} \leq \frac{1}{2}\frac{2\beta+\alpha-2}{\beta+\alpha-1}$.

Inner Bound: inspired by the LDA scheme in Figs. 2(a) and 3(a), it is possible to show that the following rate, with $\gamma' = \frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x} \overset{\mathsf{SNR}\gg 1}{\to} \gamma^\star$, with $x := \log\left(1+\frac{\mathsf{SI}}{1+\mathsf{S}+\mathsf{I}}\right) \overset{\mathsf{SNR}\gg 1}{\to} \min\{1,\alpha\} = \alpha$, is achievable

$$
\begin{aligned}
&(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = \gamma' \log\left(1+\mathsf{S}\right) - \gamma' \log\left(1+\frac{\mathsf{S}}{1+\mathsf{S}}\right) + \gamma' \log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right) \\
&+\left(1-\gamma'\right)\log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}\right) + \left(1-\gamma'\right)\log\left(1+\mathsf{S}+\frac{\mathsf{I}}{1+\mathsf{I}}\right) - \left(1-\gamma'\right)\log\left(1+\frac{\mathsf{I}}{1+\mathsf{I}}\right). \quad (17)
\end{aligned}
$$

The sum-rate in (17) implies $2\mathsf{d} = \frac{\alpha}{\beta+\alpha-1}[1-0+(\beta-1)] + \frac{\beta-1}{\beta+\alpha-1}[(1-\alpha)+1-0] = \frac{2\beta+\alpha-2}{\beta+\alpha-1}$. Thus, the gDoF upper bound is achievable.

Outer Bound: the tightest sum-rate outer bound is that in (5) (with $\mathsf{I_p} = 0$). With the first constraint in (5), $\mathsf{GAP} \leq 4$ bits; with the second one in (5), $\mathsf{GAP} \leq 5$ bits. Thus, $\mathsf{GAP} \leq 5$ bits.

APPENDIX D

ACHIEVABLE SCHEMES

Here we develop achievable schemes inspired by Fig. 2. The transmission is divided into two phases, where the first phase is the same for all regions. In the following all signal X_{b_j} for some subscript j, are independent proper-complex Gaussian random variables with zero mean and unit variance and represent codebooks used to convey the bits in b_j.

A. *Phase 1 for all cases – see also Fig. 2(a)*

Phase 1 is of duration γ. The transmit signals are

$$\mathsf{X}_{\mathsf{p}}[1] = \sqrt{1-\eta}\mathsf{X}_{b_1[1]} + \sqrt{\eta}\mathsf{X}_{b_2}, \quad \mathsf{X}_{\mathsf{c}}[1] = 0, \quad \eta := \frac{1}{1+\mathsf{S}}.$$

CTx applies successive decoding of $\mathsf{X}_{b_1[1]}$ followed by X_{b_2} from

$$\mathsf{Y}_{\mathsf{f}}[1] = \sqrt{\mathsf{C}}\sqrt{1-\eta}\mathsf{X}_{b_1[1]} + \sqrt{\mathsf{C}}\sqrt{\eta}\mathsf{X}_{b_2} + \mathsf{Z}_{\mathsf{f}}[1],$$

which is possible if

$$\begin{aligned} R_{b_1[1]} &\leq \gamma \log(1+\mathsf{C}) - \gamma \log\left(1 + \frac{\mathsf{C}}{1+\mathsf{S}}\right) \\ R_{b_2} &\leq \gamma \log\left(1 + \frac{\mathsf{C}}{1+\mathsf{S}}\right). \end{aligned} \tag{18}$$

PRx decodes $\mathsf{X}_{b_1[1]}$ treating X_{b_2} as noise from

$$\mathsf{Y}_{\mathsf{p}}[1] = \sqrt{\mathsf{S}}\sqrt{1-\eta}\mathsf{X}_{b_1[1]} + \sqrt{\mathsf{S}}\sqrt{\eta}\mathsf{X}_{b_2} + \mathsf{Z}_{\mathsf{p}}[1],$$

which is possible if

$$R_{b_1[1]} \leq \gamma \log(1+\mathsf{S}) - \gamma \log\left(1 + \frac{\mathsf{S}}{1+\mathsf{S}}\right). \tag{19}$$

Finally, since $\mathsf{C} > \mathsf{S}$, Phase 1 is successful if (18) and (19) are satisfied. Notice that it is possible to bound $\gamma \log\left(1 + \frac{\mathsf{S}}{1+\mathsf{S}}\right) \leq 1$ bit, which will contribute to the gap by at most 1 bit.

B. *Phase 2 for Region 3 in Fig. 4(a) – see also Fig. 2(d)*

Phase 2 is of of duration $(1-\gamma)$. The transmit signals are

$$\mathsf{X}_{\mathsf{p}}[2] = \mathsf{X}_{b_1[2]}, \quad \mathsf{X}_{\mathsf{c}}[2] = \sqrt{\eta}\mathsf{X}_{b_2} + \sqrt{1-\eta}\mathsf{X}_{b_3}, \quad \eta = \frac{1}{1+\mathsf{S}}.$$

PRx applies successive decoding in the following order: X_{b_3}, X_{b_2} and $\mathsf{X}_{b_1[2]}$ from

$$\mathsf{Y}_{\mathsf{p}}[2] = \sqrt{\mathsf{S}}\mathsf{X}_{b_1[2]} + \sqrt{\mathsf{I}}\, \mathrm{e}^{\mathrm{j}\theta_{\mathsf{p}}}\sqrt{\eta}\mathsf{X}_{b_2} + \sqrt{\mathsf{I}}\, \mathrm{e}^{\mathrm{j}\theta_{\mathsf{p}}}\sqrt{1-\eta}\mathsf{X}_{b_3} + \mathsf{Z}_{\mathsf{p}}[2],$$

which is possible if

$$R_{b_3} \le (1-\gamma)\log(1+\mathsf{S}+\mathsf{I}) - (1-\gamma)\log\left(1+\frac{\mathsf{I}}{1+\mathsf{S}}+\mathsf{S}\right), \tag{20}$$

$$R_{b_2} \le (1-\gamma)\log\left(1+\frac{\mathsf{I}}{(1+\mathsf{S})^2}\right), \tag{21}$$

$$R_{b_1[2]} \le (1-\gamma)\log(1+\mathsf{S}). \tag{22}$$

CRx successively decodes $\mathsf{X}_{b_1[2]}$ and X_{b_3} (by treating X_{b_2} as noise) from

$$\mathsf{Y_c}[2] = \sqrt{\mathsf{I}}\ \mathrm{e}^{\mathrm{j}\theta_\mathsf{c}}\mathsf{X}_{b_1[2]} + \sqrt{\mathsf{S}}\sqrt{\eta}\mathsf{X}_{b_2} + \sqrt{\mathsf{S}}\sqrt{1-\eta}\mathsf{X}_{b_3} + \mathsf{Z_c}[2],$$

which is possible if

$$R_{b_1[2]} \le (1-\gamma)\log(1+\mathsf{S}+\mathsf{I}) - (1-\gamma)\log(1+\mathsf{S}) \tag{23}$$

$$R_{b_3} \le (1-\gamma)\log(1+\mathsf{S}) - (1-\gamma)\log\left(1+\frac{\mathsf{S}}{1+\mathsf{S}}\right) \tag{24}$$

Phase 2 is successful if $\min\{$eq.(22),eq.(23)$\}$ = eq.(22), $\min\{$eq.(24),eq.(20)$\}$ = eq.(24) and eq.(21), are satisfied. By imposing that R_{b_2} is the same in both phases, that is, that (18) and (21) are equal, we get that γ should due chosen equal to γ'

$$\gamma' = \frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x}, \quad x := \log\left(1+\frac{\mathsf{I}}{(1+\mathsf{S})^2}\right).$$

Therefore the total sum-rate decoded at PRx and CRx through the two phases is $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = R_{b_1[1]} + R_{b_1[2]} + R_{b_2} + R_{b_3}$ as given in (13).

C. Phase 2 for Region 8 in Fig. 4(a) – see also Fig. 2(c)

Phase 2 is of duration $(1-\gamma)$. The transmit signals are

$$\mathsf{X_p}[2] = \sqrt{\delta_1}\mathsf{X}_{b_{1c}} + \sqrt{\delta_2}\mathsf{X}_{b_2} + \sqrt{\delta_3}\mathsf{X}_{b_{1p}}, \quad \delta_1 = 1-\delta_2-\delta_3, \quad \delta_2 = \frac{\mathsf{S}}{(1+\mathsf{I})^2}, \quad \delta_3 = \frac{1}{1+\mathsf{I}}$$

$\mathsf{X_c}[2] = \sqrt{\delta_3}\mathsf{X}_{b_{3c}} + \sqrt{1-\delta_3}\mathsf{X}_{b_{3p}}$, where $\mathsf{X}_{b_{3p}}$ is DPC-ed against X_{b_2} at $\mathsf{Y_p}[2]$.

The signal received at PRx and CRx are

$$\mathsf{Y_p}[2] = \sqrt{\mathsf{S}}\sqrt{\delta_1}\mathsf{X}_{b_{1c}} + \sqrt{\mathsf{S}}\sqrt{\delta_2}\mathsf{X}_{b_2} + \sqrt{\mathsf{S}}\sqrt{\delta_3}\mathsf{X}_{b_{1p}} + \sqrt{\mathsf{I}}\mathrm{e}^{\mathrm{j}\theta_\mathsf{p}}\sqrt{\delta_3}\mathsf{X}_{b_{3c}} + \sqrt{\mathsf{I}}\mathrm{e}^{\mathrm{j}\theta_\mathsf{p}}\sqrt{1-\delta_3}\mathsf{X}_{b_{3p}} + \mathsf{Z_p}[2],$$
$$\mathsf{Y_c}[2] = \sqrt{\mathsf{I}}\mathrm{e}^{\mathrm{j}\theta_\mathsf{c}}\sqrt{\delta_1}\mathsf{X}_{b_{1c}} + \sqrt{\mathsf{I}}\mathrm{e}^{\mathrm{j}\theta_\mathsf{c}}\sqrt{\delta_2}\mathsf{X}_{b_2} + \sqrt{\mathsf{I}}\mathrm{e}^{\mathrm{j}\theta_\mathsf{c}}\sqrt{\delta_3}\mathsf{X}_{b_{1p}} + \sqrt{\mathsf{S}}\sqrt{\delta_3}\mathsf{X}_{b_{3c}} + \sqrt{\mathsf{S}}\sqrt{1-\delta_3}\mathsf{X}_{b_{3p}} + \mathsf{Z_c}[2].$$

PRx applies successive decoding in the following order: $\mathsf{X}_{b_{1c}}$, X_{b_2}, $\mathsf{X}_{b_{3c}}$ and $\mathsf{X}_{b_{1p}}$ (by treating $\mathsf{X}_{b_{3p}}$ as noise) from $\mathsf{Y_p}[2]$, which is possible if

$$\begin{aligned}
R_{b_{1c}} &\leq (1-\gamma)\log\left(1+\mathsf{S}+\mathsf{I}\right) - (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}^2+\mathsf{SI}+\mathsf{S}}{(1+\mathsf{I})^2}\right) \\
R_{b_2} &\leq (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}^2+\mathsf{SI}+\mathsf{S}}{(1+\mathsf{I})^2}\right) - (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) &(25)\\
R_{b_{3c}} &\leq (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) - (1-\gamma)\log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}+\frac{\mathsf{I}}{1+\mathsf{I}}\right) &(26)\\
R_{b_{1p}} &\leq (1-\gamma)\log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}+\frac{\mathsf{I}}{1+\mathsf{I}}\right) - (1-\gamma)\log\left(1+\frac{\mathsf{I}}{1+\mathsf{I}}\right). &(27)
\end{aligned}$$

CRx applies successive decoding in the following order: $\mathsf{X}_{b_{3c}}$, $\mathsf{X}_{b_{1c}}$ and $\mathsf{X}_{b_{3p}}$ from $\mathsf{Y_c}[2]$, which is possible if

$$\begin{aligned}
R_{b_{3c}} &\leq (1-\gamma)\log\left(1+\mathsf{I}+\mathsf{S}\right) - (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) \\
R_{b_{1c}} &\leq (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) - (1-\gamma)\log\left(1+\frac{\mathsf{SI}+\mathsf{I}+\mathsf{I}^2}{(1+\mathsf{I})^2}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) &(28)\\
R_{b_{3p}} &\leq (1-\gamma)\log\left(1+\frac{\mathsf{S}}{1+\mathsf{I}}+\frac{\mathsf{I}}{1+\mathsf{I}}\right) - (1-\gamma)\log\left(1+\frac{\mathsf{I}}{1+\mathsf{I}}\right). &(29)
\end{aligned}$$

Thus, since we are in the regime $\mathsf{I}(1+\mathsf{I})^2 < \mathsf{S}(\mathsf{S}+1)$, Phase 2 is successful if (25), (26), (27), (28) and (29) are satisfied. By imposing that R_{b_2} is the same in both phases, that is, that (18) and (25) are equal, we get that γ should due chosen equal to γ'

$$\gamma' = \frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x}, \quad x := \log\left(1+\frac{\mathsf{S}^2}{(1+\mathsf{I})^3+\mathsf{S}+\mathsf{SI}}\right).$$

With this simple scheme, the total sum-rate decoded at PRx and CRx through the two phases is $(\mathsf{R_p}+\mathsf{R_c})^{(\mathrm{IB})} = R_{b_1} + R_{b_{1c}} + R_{b_{1p}} + R_{b_2} + R_{b_{3c}} + R_{b_{3p}}$ as given in (14).

D. Phase 2 for Region 10 in Fig. 4(a) – see also Fig. 2(b)

Phase 2 is of duration $(1-\gamma)$. The transmit signals are

$$\mathsf{X}_{\mathsf{p}}[2] = \sqrt{1-\delta}\mathsf{X}_{b_2} + \sqrt{\delta}\mathsf{X}_{b_1[2]}, \quad \delta = \frac{1}{1+\mathsf{I}}, \quad \mathsf{X}_{\mathsf{c}}[2] = \mathsf{X}_{b_3}.$$

The signal received at PRx and CRx are

$$\mathsf{Y}_{\mathsf{p}}[2] = \sqrt{\mathsf{S}}\sqrt{1-\delta}\mathsf{X}_{b_2} + \sqrt{\mathsf{S}}\sqrt{\delta}\mathsf{X}_{b_1[2]} + \sqrt{\mathsf{I}}\ \mathrm{e}^{\mathrm{j}\theta_{\mathsf{p}}}\mathsf{X}_{b_3} + \mathsf{Z}_{\mathsf{p}}[2],$$
$$\mathsf{Y}_{\mathsf{c}}[2] = \sqrt{\mathsf{I}}\ \mathrm{e}^{\mathrm{j}\theta_{\mathsf{c}}}\sqrt{1-\delta}\mathsf{X}_{b_2} + \sqrt{\mathsf{I}}\ \mathrm{e}^{\mathrm{j}\theta_{\mathsf{c}}}\sqrt{\delta}\mathsf{X}_{b_1[2]} + \sqrt{\mathsf{S}}\mathsf{X}_{b_3} + \mathsf{Z}_{\mathsf{c}}[2].$$

CTx encodes the message X_{b_3} using DPC by treating the codewords X_{b_2} as “known interference”. PRx decodes X_{b_2} and $\mathsf{X}_{b_1[2]}$ in this order (by treating X_{b_3} as noise) from $\mathsf{Y}_{\mathsf{p}}[2]$, which is possible if

$$R_{b_2} \leq (1-\gamma)\log\left(1+\mathsf{S}+\mathsf{I}\right) - (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) \tag{30}$$
$$R_{b_1[2]} \leq (1-\gamma)\log\left(1+\mathsf{I}+\frac{\mathsf{S}}{1+\mathsf{I}}\right) - (1-\gamma)\log\left(1+\mathsf{I}\right). \tag{31}$$

CRx decodes X_{b_3} (by treating $\mathsf{X}_{b_1[2]}$ as noise) from $\mathsf{Y}_{\mathsf{c}}[2]$, which is possible if

$$R_{b_3} \leq (1-\gamma)\log\left(1+\mathsf{S}+\frac{\mathsf{I}}{1+\mathsf{I}}\right) - (1-\gamma)\log\left(1+\frac{\mathsf{I}}{1+\mathsf{I}}\right). \tag{32}$$

Thus Phase 2 is successful if (30), (31) and (32) are satisfied. By imposing that R_{b_2} is the same in both phases, that is, that (18) and (30) are equal, we get that γ should due chosen equal to γ'

$$\gamma' = \frac{x}{\log\left(1+\frac{\mathsf{C}}{1+\mathsf{S}}\right)+x}, \quad x := \log\left(1+\frac{\mathsf{SI}}{(1+\mathsf{I})^2+\mathsf{S}}\right).$$

With this simple scheme, the total sum-rate decoded at PRx and CRx through the two phases is $(\mathsf{R}_{\mathsf{p}}+\mathsf{R}_{\mathsf{c}})^{(\mathrm{IB})} = R_{b_1[1]} + R_{b_1[2]} + R_{b_2} + R_{b_3}$ as given in (15).

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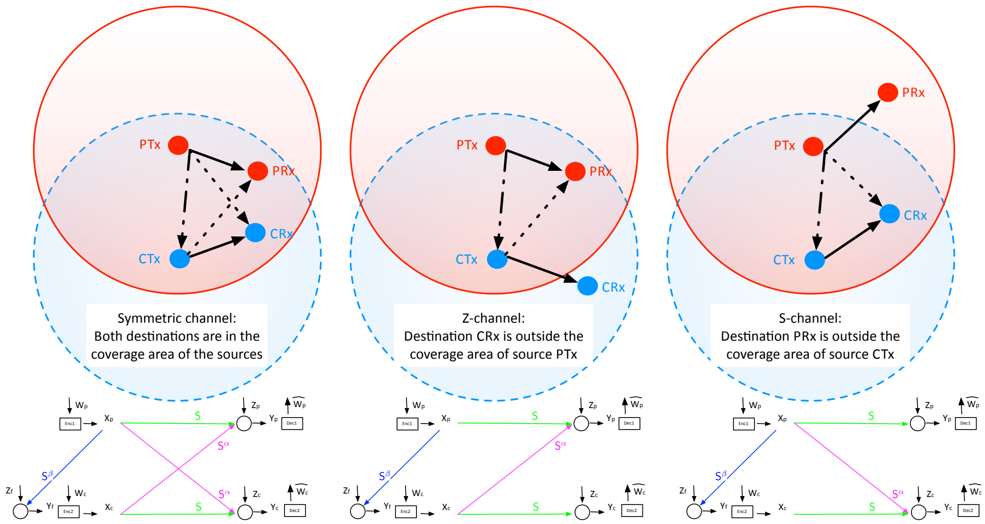
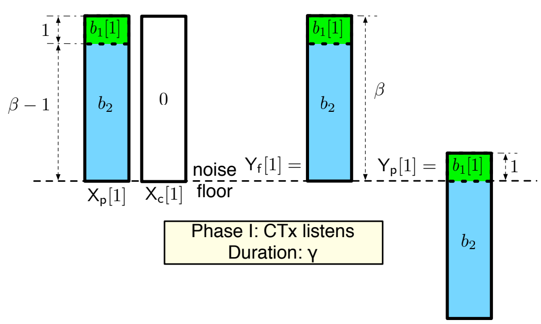
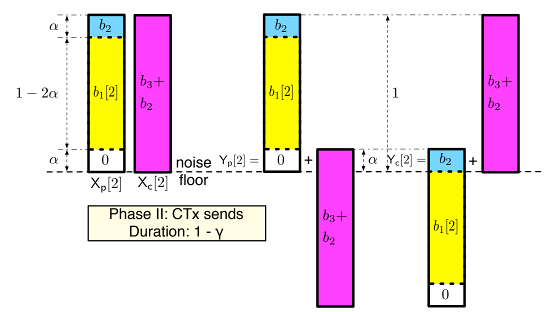


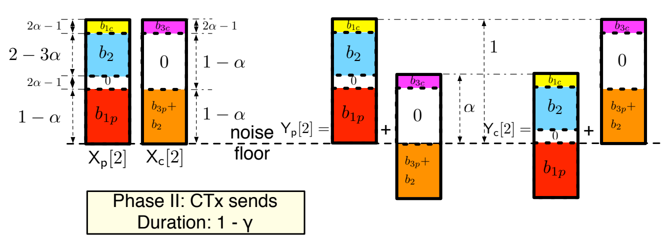
Fig. 1: The G-HD-CCIC. Left: symmetric channel. Center: Z-channel. Right: S-channel.



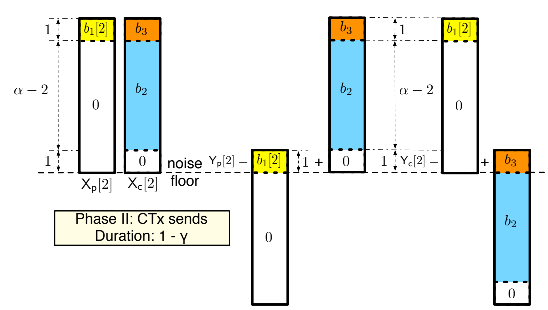
(a) Phase 1 ($\mathsf{M_c} = 0$).



(b) Phase 2 ($\mathsf{M_c} = 1$) for $\alpha \in [0, 1/2)$.

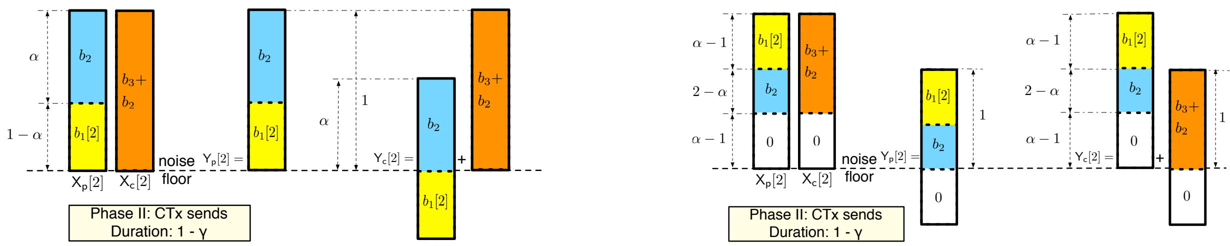


(c) Phase 2 ($\mathsf{M_c} = 1$) for $\alpha \in [1/2, 2/3)$.



(d) Phase 2 ($\mathsf{M_c} = 1$) for $\alpha \in [2, \infty)$.

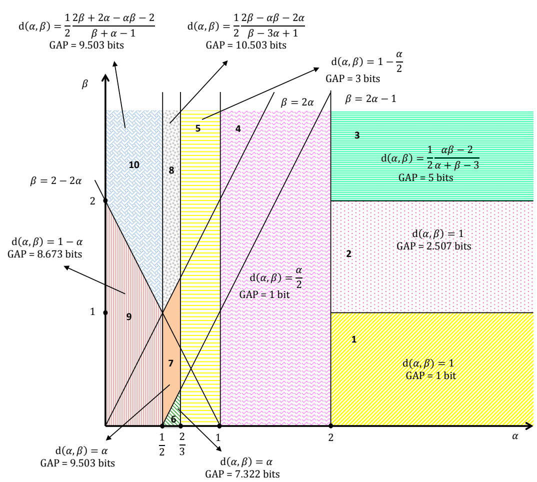
Fig. 2: Achievable strategies for the symmetric LDA HD-CCIC.



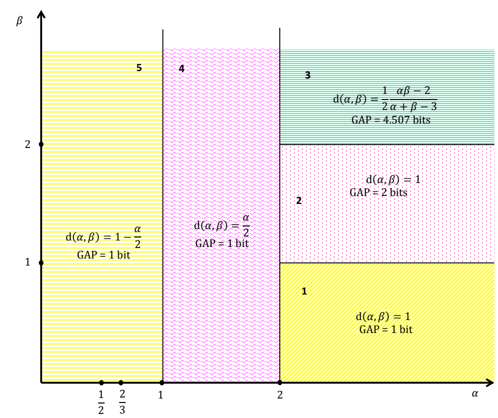
(a) Phase 2 ($\mathsf{M_c} = 1$) for $\alpha \in [0, 1)$.

(b) Phase 2 ($\mathsf{M_c} = 1$) for $\alpha \in [1, 2)$.

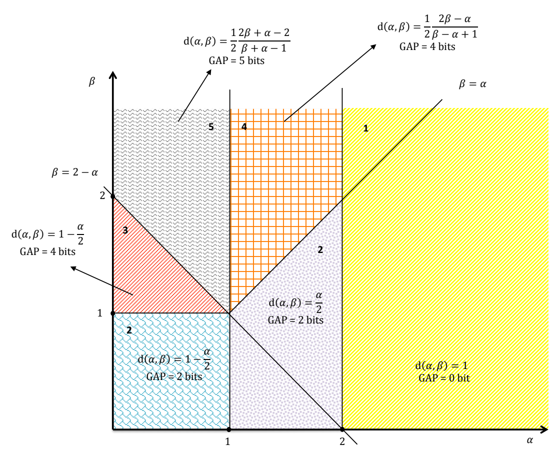
Fig. 3: Achievable strategies for the asymmetric / S-channel LDA HD-CCIC.



(a) Interference-symmetric G-HD-CCIC.



(b) Z-channel.



(c) S-channel.

Fig. 4: gDoF and constant gap for the symmetric- and asymmetric-interference G-HD-CCIC.